

Comparing OWA and WOWA filters in mean SAR images

L. Torres, J. C. Becceneri, C. C. Freitas, S. J. S. Sant'Anna, S. Sandri

¹ Instituto Nacional de Pesquisas Espaciais – INPE
Caixa Postal 515 – 12227-010 – São José dos Campos - SP, Brazil
ljmtorres@gmail.com,
jose.becceneri, sandra.sandri@inpe.br,
corina, sidnei@dpi.inpe.br

Abstract. We compare WM, OWA and WOWA filters in mean SAR images, respectively based on the parameterized families of operators WM (Weighted Means), OWA (Ordered Weighted Average) and WOWA (Weighted OWA). The WM and OWA operators use a single vector to weight an input data vector: one whose weights correspond to the same positions in the input, and another that considers the ordered positions of the input, respectively. WOWA operators, however, use both of these weight vectors. We present an application on SAR imagery, using simulated images derived from a real-world scene and 5×5 windows, in which we make use of a Genetic Algorithm to tune the parameters of these families of operators.

Keywords: OWA filters, WOWA filters, SAR images, Speckle.

1. Introduction

The Weighted Mean operators (WM) attain a convex combination of a set of values, using a weight vector \mathbf{p} , whose weights dimension the significance of the data with independence of the value that the source has captured. The Ordered Weighted Average operators (OWA) were introduced by Yager (1988), in the Fuzzy Sets Theory domain. This family of mean operators produce a convex arrangement of a set of ordered values, using a weight vector \mathbf{w} . In OWA, contrarily to WM, the weights measure the importance of a value (in relation to other values) with independence of the source that has captured it (TORRA, 1997). Another important family of mean operators is the Weighted OWA (WOWA) operators, proposed by Torra (1997), that use both vectors \mathbf{p} and \mathbf{w} to weight data, and aims to take advantage of the gains of both OWA and WM operators.

In previous works Torres et al. (2015, 2016a), we introduced OWA filters, that use OWA operators for data filtering. In these papers, we explored some strategies to learn vector \mathbf{w} in OWA filters to reduce speckle in SAR imagery, using Genetic Algorithms (GA) (HOLLAND, 1975). In both works we only dealt with intensity images; in Torres et al. (2015) we addressed a single polarization (HH) and 3×3 windows, whereas in Torres et al. (2016a) we addressed three polarizations (HH, HV and VV) and 5×5 windows. Recently, we introduced WOWA filters (TORRES et al., 2016b), based on WOWA operators, and addressed three strategies to learn the weight vectors using GAs, employing only the HH polarization and 3×3 windows.

This work proposes a new study with the application of the WM, OWA and WOWA filters for speckle noise reduction in SAR imagery, in polarizations HH, HV and VV. We use 5×5 windows and the best strategy for learning WOWA filters, according to the experiments in Torres et al. (2016b). As in the works with OWA and WOWA filters, we use a fragment of a phantom described in Saldanha (2013), with the synthetic images for the polarizations simulated using the parameters for Wishart distributions estimated in Silva et al. (2013) from a real-world scene. We have also compared the results of our filters with those issued by two model-dependent filters proposed in Lee et al. (2006) and Torres et al. (2014).

2. Basic concepts on SAR images

Synthetic Aperture Radar (SAR) data are generated by a system of coherent illumination and are affected by the coherent interference of the signal. It is known that these data incorporate a granular noise that degrades its quality, known as speckle noise, which is also present in the laser, ultrasound-B, and sonar imagery (LEE; POTTIER, 2009). The noise makes it a hard task

to obtain the segmentation, extraction, analysis and, classification of objects and information in SAR images.

SAR systems generate the image of a target area by moving along a usually linear trajectory, and transmitting pulses in lateral looks towards the ground, in either horizontal or vertical polarizations (RICHARDS, 2009), respectively denoted as H and V. In most imaging radars, the bands use frequencies in the 2MHz to 12.5GHz range, with wavelengths between 2.4cm and 1m.

The reception of the transmitted energy used to be made on the same polarization of the transmission only, generating images in the HH and VV polarizations. Nowadays, with the advent of polarimetric or polarized radars (PolSAR), images relating to HV and VH polarizations are also obtained, using information about intensity and phase of the cross signals.

A complex image is generated for each polarization from a given a scene, with the real and imaginary components for each pixel. The complex images from HH, VV, and HV polarizations are denoted as S_{HH} , S_{HV} , and S_{VV} . Multiplying the vector $[S_{HH} \ S_{HV} \ S_{VV}]$ by its transposed conjugated vector $[S_{HH}^* \ S_{HV}^* \ S_{VV}^*]^t$, a 3×3 covariance matrix is obtained. The three images in the main diagonal, denoted by I_{HH} , I_{HV} , and I_{VV} , contain intensity values.

2.1 Filters for SAR imagery

According to Lee and Pottier (2009), SAR image filtering requires preserving the target response. Such requirement can be posed as: (i) each element of the image should be filtered in a similar way to multilook processing by averaging the data of neighboring pixels; and (ii) homogeneous regions in the neighborhood should be adaptively selected to preserve resolution, edges and the image quality. The second requirement, i.e. selecting homogeneous areas given similarity criterion, is a common problem in pattern recognition and artificial intelligence. It boils down to identifying observations from different stationary processes.

The simplest filters are linear filters that employ the convolution operation, described as follows. Given an image I , whose pixels take values in R , a $m \times m$ window around the central pixel (x, y) in I , and a matrix of coefficients $\gamma : \{-m, \dots, 0, \dots, m\}^2 \rightarrow R$, the result of convolution is a filtered image I_γ , calculated as

$$I_\gamma(x, y) = \sum_{i=-m, m} \sum_{j=-m, m} \gamma(i, j) \times I(x + i, y + j).$$

Order Statistics Filters (BOVIK et al., 2005) are a general class of filters in which the result of filtering for a given pixel is the linear combination of the ordered values of the pixels in the window around that pixel. These filters belong to the larger class of non-linear filters based on order statistics (PITAS; VENETSANOPOULOS, 2013), an application of L-estimators. An OSF is obtained when a convolution filter is applied on the ordered statistic of the pixel values in a window.

More complex filters are obtained with the adoption of a model to the noise. The so-called Lee filter is one of such filters. In this filter, speckle reduction is based on multiplicative noise model using the minimum mean-square error (MMSE) criterion (LEE et al., 1991, 1999). An improved version of the Lee filter, known as the Refined Lee filter (LEE et al., 2006), here called R-Lee filter, uses a methodology for selecting neighboring pixels with similar scattering characteristics.

Another model-dependent filter is the Nonlocal Means (NL-means) Buades et al. (2005), which uses similarities between patches as the weights of a mean filter, and is known to decrease additive Gaussian noise. A more recent filter, the Stochastic Distances and Nonlocal Means filter (SDNLM) (TORRES et al., 2014), is an adaptive nonlinear extension of the NL-means algorithm filter. In SDNLM, overlapping samples are compared based on stochastic distances

between distributions, and the p -values resulting from such comparisons are used to build the weights of an adaptive linear filter.

2.2 Image Quality Assessment for SAR imagery

According to Wang et al. (2002) image quality assessment in general, and filter performance evaluation in particular, are hard tasks and crucial for most image processing applications. Two important indices used on quality assessment of filtered images are NMSE and SSIM, described below.

The NMSE (Normalized Mean Square Error) index is a general purpose error measure, widely employed in image processing (see Baxter and Seibert (1998)). Let r be the perfect information data and s an approximation of r ; NMSE is calculated as:

$$NMSE = \frac{\sum_{j=1}^n (r_j - s_j)^2}{\sum_{j=1}^n r_j^2}, \quad (1)$$

where r_j and s_j refer to values in r and s at the same coordinates (the position of a given pixel in the case of images). NMSE always yield positive values, and the lower its value, the better is the approximation considered to be.

The SSIM (Structural SIMilarity) index is an improved version of the universal image quality index proposed proposed by Wang and Bovik (2002). SSIM measures the similarity between two scalar-valued images; it can be viewed as a quality measure of one of the images, when the other image is regarded as of perfect quality (WANG et al., 2004). SSIM takes into account three factors: (i) correlation between edges; (ii) brightness distortion; and (iii) distortion contrast. Let r and s be the perfect information and its approximation, respectively; SSIM is calculated as

$$SSIM(r, s) = \frac{Cov(r, s) + \alpha_1}{\hat{\sigma}_r \hat{\sigma}_s + \alpha_1} \times \frac{2\bar{r}\bar{s} + \alpha_2}{\bar{r}^2 + \bar{s}^2 + \alpha_2} \times \frac{2\hat{\sigma}_r \hat{\sigma}_s + \alpha_3}{\hat{\sigma}_r^2 + \hat{\sigma}_s^2 + \alpha_3}, \quad (2)$$

where \bar{r} and \bar{s} are sample means, $\hat{\sigma}_r^2$ and $\hat{\sigma}_s^2$ are the sample variances, $Cov(r, s)$ is the sample covariance between r and s , and constants α_1 , α_2 and α_3 are used the index stabilization. SSIM ranges in the $[-1, 1]$ interval, and the higher its value, the better is the approximation considered to be.

3. WM, OWA and WOWA operators

Let \mathbf{p} be a weighting vector of dimension n ($\mathbf{p} = [p_1 \ p_2 \ \dots \ p_n]$), such that:

- (i) $p_i \in [0, 1]$;
- (ii) $\sum_i p_i = 1$.

A mapping $f_p^{wm} : R^n \rightarrow R$ is a Weighted Mean Operator (WM) of dimension n , associated to p , if:

$$f_p^{wm}(a_1, \dots, a_n) = \sum_i p_i \times a_i. \quad (3)$$

The Ordered Weighted Average operators (YAGER, 1988) and the Weighted OWA operators (TORRA, 1997) are important families of aggregation operators, proposed in the context of Fuzzy Sets Theory.

Let \mathbf{w} be a weighting vector of dimension n ($\mathbf{w} = [w_1 \ w_2 \ \dots \ w_n]$), such that:

- (i) $w_i \in [0, 1]$;
- (ii) $\sum_i w_i = 1$.

A mapping $f_w^{owa} : R^n \rightarrow R$ is an Ordered Weighted Average Operator (OWA) of dimension n , associated to w , if (YAGER, 1988):

$$f_w^{owa}(a_1, \dots, a_n) = \sum_i w_i \times a_{\sigma(i)}, \quad (4)$$

where $\{\sigma(1), \dots, \sigma(n)\}$ is a permutation of $\{1, \dots, n\}$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$, for all $i = \{2, \dots, n\}$ (i.e., $a_{\sigma(i)}$ is the i -th largest element in $\{a_1, \dots, a_n\}$).

Some well-known OWA operators are the mean, min, max and median, which are obtained with OWA vectors w_{mean} , w_{min} , w_{max} , and w_{med} , respectively. For $n = 3$, we have: $w_{mean} = [1/3, 1/3, 1/3]$, $w_{min} = [0, 0, 1]$, $w_{max} = [1, 0, 0]$, and $w_{med} = [0, 1, 0]$.

Let \mathbf{p} and \mathbf{w} be weighting vectors as given above. A mapping $f_{w,p}^{wowa} : R^n \rightarrow R$ is a Weighted Ordered Weighted Average (WOWA) operator of dimension n , associated to \mathbf{p} and \mathbf{w} , if (TORRA, 1997):

$$f_{w,p,\phi}^{wowa}(a_1, \dots, a_n) = \sum_i \omega_i \times a_{\sigma(i)}, \quad (5)$$

where $\{\sigma(1), \dots, \sigma(n)\}$ is a permutation of $\{1, \dots, n\}$, for all $i = \{2, \dots, n\}$, such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$, weight ω_i is defined as

$$\omega_i = \phi(P_\sigma(i)) - \phi(P_\sigma(i-1)), \quad (6)$$

$$P_\sigma(i) = \sum_{j \leq i} p_{\sigma(j)}, \quad (7)$$

and ϕ is a monotone increasing function that interpolates points $(0, 0)$ and $(i/n, \sum_{j \leq i} w_j)$, $i = 1, n$. Function ϕ is required to be a straight line when the points can be interpolated in a linear way. Torra (1997) proves that the ω_i 's compose a weighting vector of dimension n ($\omega = [\omega_1 \dots \omega_n]$), such that:

- (i) $\omega_i \in [0, 1]$;
- (ii) $\sum_i \omega_i = 1$.

In Torra (1997), we can also find examples of non-linear functions to implement ϕ and the proof that OWA and WM are particular cases of WOWA operators.

Some well-known WM operators are the mean, min, max. These operators, along with the median, are also OWA and WOWA operators. We obtain these OWA operators using OWA vectors w_{mean} , w_{min} , w_{max} , and w_{med} , which, for $n = 3$, are given respectively as $w_{mean} = [1/3, 1/3, 1/3]$, $w_{min} = [0, 0, 1]$, $w_{max} = [1, 0, 0]$, and $w_{med} = [0, 1, 0]$.

4. WM, OWA and WOWA filters for mean images

In (TORRES et al., 2015, 2016a), we introduced OWA filters for images, whereas in Torres et al. (2016b), we introduced WOWA filters. OWA and WOWA filters consist in applying OWA weight vectors in the values inside a sliding window over a given image. Below, we describe WOWA filters $F_{w,p,\phi}^{wowa}$, defined in Torres et al. (2016b), to obtain a filtered image $I_{w,p,\phi}^{wowa}$ from an image I in 3 polarizations.

Procedure $F_{w,p,\phi}^{wowa}(I)$ for mean images

1. Obtain the mean image II from a given image I , taking the simple average of the pixel values of the polarizations HH, HV and VV from I .
2. Transform a weight matrix \mathbf{M} associated to a predefined neighborhood, into a vector with n positions \mathbf{p} .
3. For each pixel in position (x, y) in image II , transform a window II' around (x, y) , according to the predefined neighborhood, into a vector of n positions \mathbf{a} .
4. Using \mathbf{a} , derive σ and \mathbf{a}_σ .
5. Using \mathbf{w}_0 , \mathbf{p}_0 and ϕ_0 , derive weight vector ω .
6. Calculate $f_{w,p,\phi}^{wowa}(a_1, \dots, a_n)$.
7. Make the result become the value for position (x, y) in the filtered image:

$$II_{w,p,\phi}^{wowa}(x, y) = f_{w,p,\phi}^{wowa}(a_1, \dots, a_n).$$

Given a position (x, y) in a mean image II and considering a 3×3 window, in step 3 we would obtain a vector \mathbf{a} , with 9 positions, as $(I(x-1, y-1), I(x-1, y), I(x-1, y+1), I(x, y-1), I(x, y), I(x, y+1), I(x+1, y-1), I(x+1, y), I(x+1, y+1))$. Vector \mathbf{p} is obtained in a similar way from \mathbf{M} in step 1. Using a 5×5 window, vector \mathbf{a} has 25 positions.

OWA and WM filters F_w^{owa} and F_p^{wm} are obtained in a similar but simpler way. Note that F_p^{wm} is a convolution filter, whereas F_w^{owa} is an OSF. A WOWA filter is a combination of convolution filters and OSFs, enjoying the advantages of both.

5. Experiments

We conducted a series of experiments using intensity SAR images in L-band with wavelengths of [30cm,1m] and frequencies of [1MHz,2GHz], in polarizations HH, HV and VV. We used a fragment of a phantom described in (SALDANHA, 2013) (see Figure 3) and a set of 50 synthetic images, simulated using the parameters for Wishart distribution estimated in (SILVA et al., 2013) for an area in the Brazilian Amazon region. Each simulated image has 240×240 pixels and was generated with 1-look.

We employed Genetic Algorithms (GAs) (HOLLAND, 1975) to learn the weight vectors. Given an image, our experiments have been performed as follows: (a) the parameters of the distributions associated to the various regions in the image are estimated, (b) a set of simulated images is randomly created using the distributions parameters, (c) the set of simulated images is partitioned in two sets, one for training and one for testing, and (d) the best weight vector found by the GA on the training set is used on the test set for evaluation.

For the GA experiments, we performed a 5-fold cross-validation, using 40 images for training and 10 for testing in each fold. We used a 5×5 window and the weight vectors \mathbf{p} and \mathbf{w} thus have 9 positions each. The elements in the initial population in each experiment were chosen at random. In (TORRES et al., 2016b) we investigated three strategies to learn the weight vectors using GAs, using only the HH polarization and 3×3 windows: i) learn vectors \mathbf{p} and \mathbf{w} at the same time, ii) learn \mathbf{p} then \mathbf{w} , and iii) learn \mathbf{w} then \mathbf{p} . Since the first strategy yielded the best results, we adopted it in the present work.

The GA was run on a machine with the following specifications: Intel i7, CPU 2.60 GHz, RAM with 16 GB, Windows 10, Fortran with Force 2.0 compiler. Considering 30 generations and 5 folds, with 10 images in each fold, and 36 elements in the population, the GA processing took around 1 hour, for no matter the value of the other parameters and operators. As expected, the largest number of generations and the largest the populations, the longer the training process takes. Also, as the number of generations doubled, so roughly did the training time. Finally, as the size of the populations doubled, the training time was gradually lower than the double.

After a series of small experiments with various alternatives, we decided to fully test the proposed procedure considering a set of parametrizations for the GA: 3 population sizes (18, 36 and 72 elements), 2 numbers of generations (10 and 30), 2 mutation rates (.2 and .8), 3 seeds for random numbers (2, 70 and 271). We used roulette as the selection mechanism in all experiments. As fitness function for each fold in each parametrization, we took the mean NMSE value of the resulting filtered images.

Considering the five folds, the best overall result was obtained using OWA operators with seed 70, 36 elements in the population, mutation rate .2 and 30 generations¹. Table 1 brings the results obtained with filters whose parameters have been learned with the GAs with seed 70. In the Table 1, we also report the results for SDNLM and R-Lee filters and main OSF filters. For SDNLM and R-Lee filters, the best parametrizations were chosen after a few experiments, using 5×5 filtering window for both filters, with 3×3 patches, and significance level of 5% for SDNLM, and with $ENL = 1$ for R-Lee.

¹In the experiments we used mutation strategy A, described in Torres et al. (2016a, 2016b).

Table 1. NMSE and SSIM mean and standard deviation for 5 folds.

	NMSE		SSIM	
	mean	std	mean	std
Simulated (no filter)	0.4317	6.08E-3	0.0396	8.00E-9
SDNLM	0.0339	8.94E-4	0.0352	1.10E-8
R-Lee	0.0460	1.14E-3	0.0378	4.00E-9
OWA	0.0283	7.32E-5	0.0134	0.00
WOWA	0.0290	6.44E-5	0.0068	4.00E-9
WM	0.0289	5.91E-5	0.0081	1.00E-9
Mean	0.0287	4.53E-4	0.0129	1.59E-3
Median	0.0549	9.68E-4	0.0130	1.47E-3
Minimum	0.6806	2.67E-3	0.0130	4.36E-4
Maximum	3.6345	8.22E-2	0.0118	3.10E-3

We see in Table 1 that the best NMSE mean results are obtained with OWA, WOWA and WM filters, followed by the mean filter, and then by SDNLM and Lee filters. For SSIM, the higher the better, and we see that a different order of quality is obtained, with the best results obtained by Lee filter, followed by and SDNLM and OWA filters. Note, however, that in this application the simulated images obtained a better SSIM than the filtered ones.

6. Conclusions and future work

We investigated WM, OWA and WOWA filters in mean images (obtained from intensity images polarizations HH, HV and VV), respectively based on the parametrized families of operators of Weighted Means, Ordered Weighted Average (OWA) and Weighted OWA, upon an application on SAR imagery, using simulated images derived from a real-world scene. We used 5×5 windows, and a Genetic Algorithm to tune the parameters of these families of operators, and the mean values of NMSE as fitness function.

We compared the obtained filters with model-based filters SDNLM (TORRES et al., 2014) and R-Lee (LEE et al., 2006), that use full polarimetric images. The best results for NMSE were obtained by WM, OWA and WOWA filters, whereas for SSIM, the model-dependent filters fared better.

In the future, we intend to study the influence of initial populations in the GA, and address multi-optimization issues to learn the best weights considering more than one quality index. We also intend to investigate the use of WM, OWA and WOWA filters with full polarimetric images.

References

- BAXTER, R.; SEIBERT, M. Synthetic aperture radar image coding. **MIT Lincoln Laboratory Journal**, v. 11, n. 2, p. 121–158, 1998.
- BOVIK, A.; HUANG, T.; MUNSON, D. A generalization of median filtering using linear combinations of order statistics. **IEEE Trans. on Acoustics, Speech and Signal Processing**, ASSP-31, n. 6, p. 1342–1349, 2005.
- BUADES, A.; COLL, B.; MOREL, J. M. A review of image denoising algorithms, with a new one. **Multiscale Modeling & Simulation**, SIAM Publications, v. 4, n. 2, p. 490–530, 2005. ISSN 1540-3459.
- HOLLAND, J. H. **Adaptation in Natural and Artificial Systems**. [S.l.]: University of Michigan Press, 1975.

- LEE, J.-S.; GRUNES, M. R.; GRANDI, G. de. Polarimetric SAR speckle filtering and its implication for classification. **IEEE Transactions on Geoscience and Remote Sensing**, v. 37, n. 5, p. 2363–2373, 1999.
- LEE, J.-S.; GRUNES, M. R.; MANGO, S. A. Speckle reduction in multipolarization, multifrequency SAR imagery. **IEEE Transactions on Geoscience and Remote Sensing**, v. 29, n. 4, p. 535–544, 1991.
- LEE, J.-S.; GRUNES, M. R.; SCHULER, D. L.; POTTIER, E.; FERRO-FAMIL, L. Scattering-model-based speckle filtering of polarimetric SAR data. **IEEE Transactions on Geoscience and Remote Sensing**, v. 44, n. 1, p. 176–187, 2006.
- LEE, J.-S.; POTTIER, E. **Polarimetric Radar Imaging: From Basics to Applications**. [S.l.]: Taylor & Francis, 2009. (Optical Science and Engineering). ISBN 9781420054989.
- PITAS, I.; VENETSANOPOULOS, A. N. **Nonlinear Digital Filters: Principles and Applications**. [S.l.]: Springer, 2013.
- RICHARDS, J. **Remote Sensing with Imaging Radar**. [S.l.]: Springer, 2009. (Signals and Communication Technology). ISBN 9783642020209.
- SALDANHA, M. F. S. **Um segmentador multinível para imagens SAR polarimétricas baseado na distribuição Wishart**. PhD Thesis (PhD Thesis) — INPE, 2013.
- SILVA, W. B.; FREITAS, C. C.; SANT’ANNA, S. J. S.; FRERY, A. C. Classification of segments in PolSAR imagery by minimum stochastic distances between Wishart distributions. **IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing**, v. 6, n. 3, p. 1263–1273, 2013.
- TORRA, V. The weighted owa operator. **International Journal of Intelligent Systems**, v. 12, n. 2, p. 153–166, 1997.
- TORRES, L.; BECCENERI, J. C.; FREITAS, C. C.; SANT’ANNA, S. J. S.; SANDRI, S. Owa filters for sar imagery. In: **La-CCP’15**. Curitiba: [s.n.], 2015. p. 1–6.
- TORRES, L.; BECCENERI, J. C.; FREITAS, C. C.; SANT’ANNA, S. J. S.; SANDRI, S. Bio-inspired computation & applications in image processing. In: _____. [S.l.]: Elsevier, 2016. chapter Learning OWA Filters parameters for SAR Imagery with multiple polarizations.
- TORRES, L.; BECCENERI, J. C.; FREITAS, C. C.; SANT’ANNA, S. J. S.; SANDRI, S. Wow image filters. In: **IV CBSF**. Campinas: [s.n.], 2016. p. 1–13.
- TORRES, L.; SANT’ANNA, S. J. S.; FREITAS, C. C.; FRERY, A. C. Speckle reduction in polarimetric SAR imagery with stochastic distances and nonlocal means. **Pattern Recognition**, v. 47, n. 1, p. 141–157, 2014.
- WANG, Z.; BOVIK, A. C. A universal image quality index. **IEEE Signal Processing Letters**, v. 9, n. 3, p. 81–84, 2002.
- WANG, Z.; BOVIK, A. C.; LU, L. Why is image quality assessment so difficult? In: **IEEE International Conference on Acoustics, Speech, & Signal Processing**. Orlando: [s.n.], 2002. v. 4, p. 3313–3316.
- WANG, Z.; BOVIK, A. C.; SHEIKH, H. R.; SIMONCELLI, E. P. Image quality assessment: From error visibility to structural similarity. **IEEE Transactions on Image Processing**, v. 13, n. 4, p. 600–612, 2004.
- YAGER, R. On ordered weighted averaging aggregation operators in multi-criteria decision making. **IEEE Transactions on Systems, Man and Cybernetics**, v. 18, p. 183–190, 1988.